EVALUATION REPORT OF THE PAPER “EXISTENCE OF STABLE ROOMMATE MATCHINGS”-KIM-SAU CHUNG

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ECON442

Spring 2023

1. What is (are) the main problem(s) dealt with in the paper?

The paper discusses the stable matching problem in the general context. Thus, it is referred to as the stable roommate matchings problem which is the general name of the matching problem. It explores why in the specific matching problem, the marriage problem, there always exists stable matchings whereas stable matchings may not exist in the general roommate’s problem.

In fact, Tan (1991) already proved the existence of stable matchings in a general roommate matchings problem in which agents have strict preferences. However, no paper has provided an existence theorem for the case in which agents have weak preferences.

1. What is (are) the solution(s) provided to that (those) problem(s)?

The paper provides a solution for the weak preferences case. The author uses the condition they call “no odd rings” because it ensures that there exist stable roommate matchings when the agents have weak preferences. This solution was not provided by the literature earlier. This condition also guarantees that the Roth-Vande Vate (1990) process converges to a stable roommate matching with probability one.

1. To what extent is (are) the solution(s) provided complete?

The solutions are actually complete in the sense that they are provided to complete other papers’ findings. Therefore, the solutions seem like they are meant to complete the missing part of the findings of the previous papers.

Bear in mind that odd rings are the one and only source of instability. Hence, this article, speaking of this concept thoroughly is complete by only doing that.

There are some incorrect examples and proofs in the article that decreases its credibility. For example, in Example 3, the author tries to exemplify a case in which RVV process gets stuck in a loop and never reaches to a stable matching. However, the example actually represents a case in which RVV converges to a stable matching, in this case to the unique stable matching, v.

1. What are the main notions employed in solving the problem dealt with and to what extent are those notions novel?

The most important notion is the “no odds ring” condition. In fact, the article is written due to the identification of this notion and its usefulness. The author uses this condition/notion in a general matching context in order to show that, given this condition, there always exists a stable roommate matching, including the case of weak preferences.

The author utilizes the activity of “satisfying the blocking pair” to seek stability from an instable matching. This is kind of like their own algorithm for finding a stable matching from an initial matching that is instable. Lemma 1 suggests how the process of finding a stable matching would occur if a preference profile has no odd rings. As long as one initiates the “algorithm” with an initial IR matching, they can reach to a stable matching. One can see how the “algorithm” proceeds within the proof of this lemma. The algorithm repeats the process of enlarging the subset of all the agents in which there are no blocking pairs. The repetition of this process over and over will eventually result in the subset of choice becoming the whole set of agents, the matching of which has no blocking pairs within.

1. Do you find the notions introduced and the method(s) employed in solving the problem set appropriate? Do you have any suggestions concerning an alternative approach?

The notions introduced are few but very important. As mentioned before, the concept of “no odd rings” is expressed very well yet it is a very abstract concept when it comes to providing an economic intuition for it.

As the author confesses, the “no odd rings” condition is not very economically interpretable. As a result, it is hard to explain the intuition of why this condition is a game-changer in the context of stable matchings.

1. Does the paper contain any exemplary applications, which the findings of the paper are useful for? Can you think of any other applications in the light that the paper sheds on the problem it deals with?

The paper provides a general solution for matching problems. Therefore, any matching problem with the mentioned rules in the paper may find this paper’s findings useful as long as the “general matching rules” apply.

Specifically, the author applies Theorem 1 of “no odd rings” condition but tries to make the applications more interpretable through the inclusion of other sufficient conditions that are more economically interpretable.

The mentioned concept is the “tie-breaking” version of a given preference profile. The following fact is very useful in the query of stability of a matching: if a tie-breaking version of a preference profile admits stability, then the original preference profile does so as well. The concept of “tie-breaking” version of a preference profile is a very simple trick that alters some of the strict preferences to indifference, which would not doctor the possibility of stability if there exist any. This version can be used when one cannot directly prove that the original preference profile is odd-ring-free.

One application of Theorem 1 was aimed at reproving what Bartholdi and Trick (1986) theorized: if the agents of a matching problem can be demonstrated in a metric space and if each strictly prefers agents with similar preferences over agents with dissimilar preferences relative to them as roommates, and if they prefer sharing room with anyone whosoever over being alone, there exist(s) stable roommate matching(s).

1. What are the open problems that the paper leads to? Can you think of any further open problems not mentioned in the paper?

I believe the economic interpretation of the “no odd rings” is necessary to emphasize its value for matching problems. One may try to interpret its meaning in special cases of matching problems if one cannot identify any general interpretation of it over the general context of matching.

The part of the article in which “change of sex” is discussed does not make so much sense to me. According to the author, if an agent is observed twice in a sequence, that implies change of sex. From my point of view, if an agent is observed twice or more, this just implies the agent has two or more partners. In other words, that agent is in an open relationship. This concept would create a whole new set of possibilities in terms of matching and thereby should not be considered in the proof of the existence of stable matchings when agents have weak preferences, because it has nothing to do with it and it is a digression from what is to be mainly shown in the paper.

As long as one has an odd number of agents in a matching problem and they ensure that no agents appear twice or more in the chain, the “no odds ring” condition is satisfied -given the agent preferences in the article-. When open relationships are taken into consideration, everything is to be adjusted accordingly. I opine that “inclusion of open relationships” along with couple of assumptions would be an interesting open problem to ruminate on.

1. What is your overall evaluation concerning the impact of the paper on the research field it belongs to?

Stability is an important concept for matching problems, and this paper provides a condition for all of the matching problems that would ensure stability. Note that stability’s definition mildly debatable. However, based on its current definition, it seems to be an important concept for matching problems thus this paper with its findings must have a significant impact on the research field it belongs to.

1. Are there any other questions not included in the above list that you consider relevant for an evaluation of a research paper, and a what are your answers to those questions in the context of the paper you are assigned to?

Additional Comments:

The paper provides a very good literature view and a preliminaries section. However, the preliminaries section also makes the paper look like it has a lot to say even though it does not (quantitively, because qualitatively it does have a lot to say). Academics in the Graph Theory may find this approach inefficient and misleading.

However, for an undergraduate student, this is perfect for recalling microeconomic concepts like “preference profile”, “Individiual Rationality”.

One hardship is that the papers reviewed in this paper have complex concepts utilized within them, and this paper tries to build on or refers to those concepts such as “Roth-Vande Vate Process” (RVV) along with “random-paths-to-stability”, and “fractional stable matching polytope”.

The question is indeed an interesting one: Why does the marriage problem, a special case of matching problem, stand out as the problem that always has a stable matching.

It is great that the author contextualizes matching problems as “decentralized decision making processes”. This makes matching problems seem as more realistic problems, rather than complicated fantasies.

The author builds on and criticizes Roth and Vande Vate (1990). First, they narrate what the paper has proved: the convergence of any decentralized decision process to stability under strict preferences. Then, they emphasize two problems that one would encounter if the method in this paper is used elsewhere (e.g., weak preferences case). The first one is the fact that a stable matching may not exist at all, thereby there is nothing to converge to. Secondly, the RVV process may end up in a loop that would never arrive to a stable matching.

Upon identifying these difficulties, especially the second one, the author tries to identify a condition that would make RVV process always converge to a stable roommate matching.